Bayesian Networks in Credit Rating

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Bayesian Networks in Credit Rating

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- Bayesian Networks
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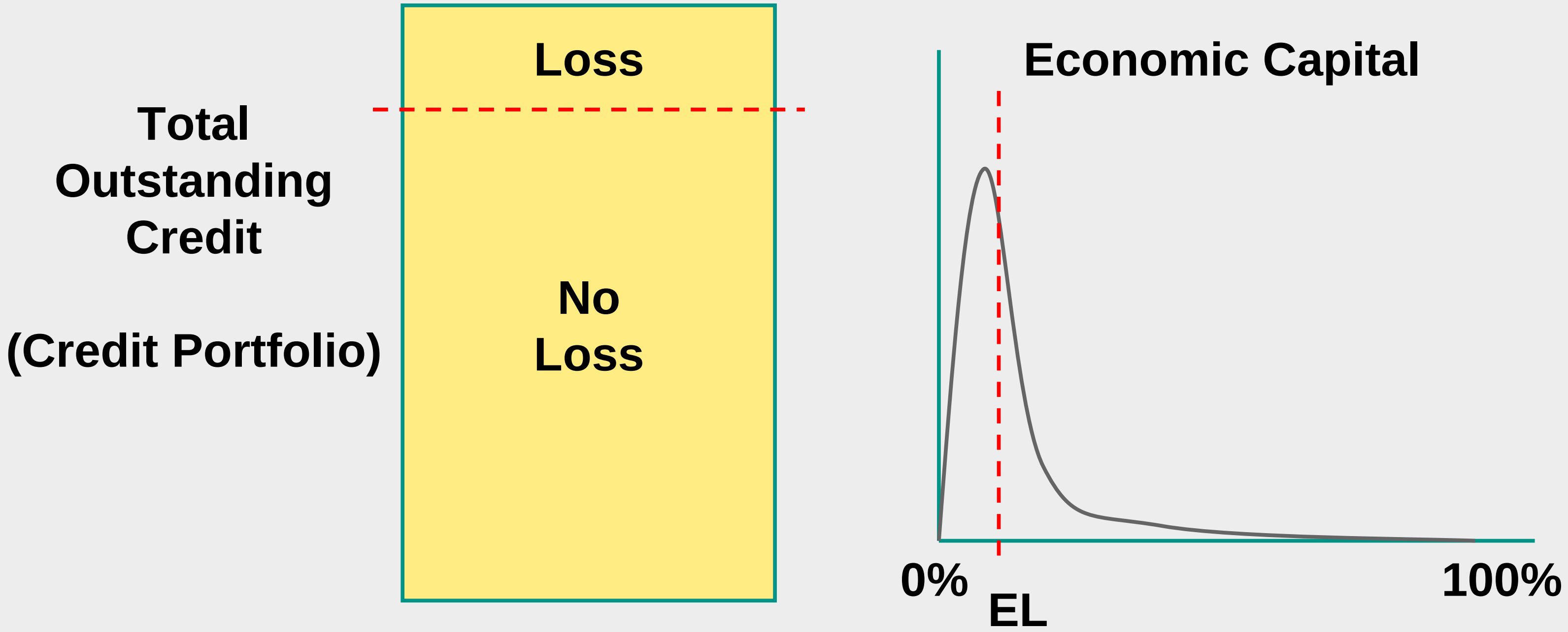
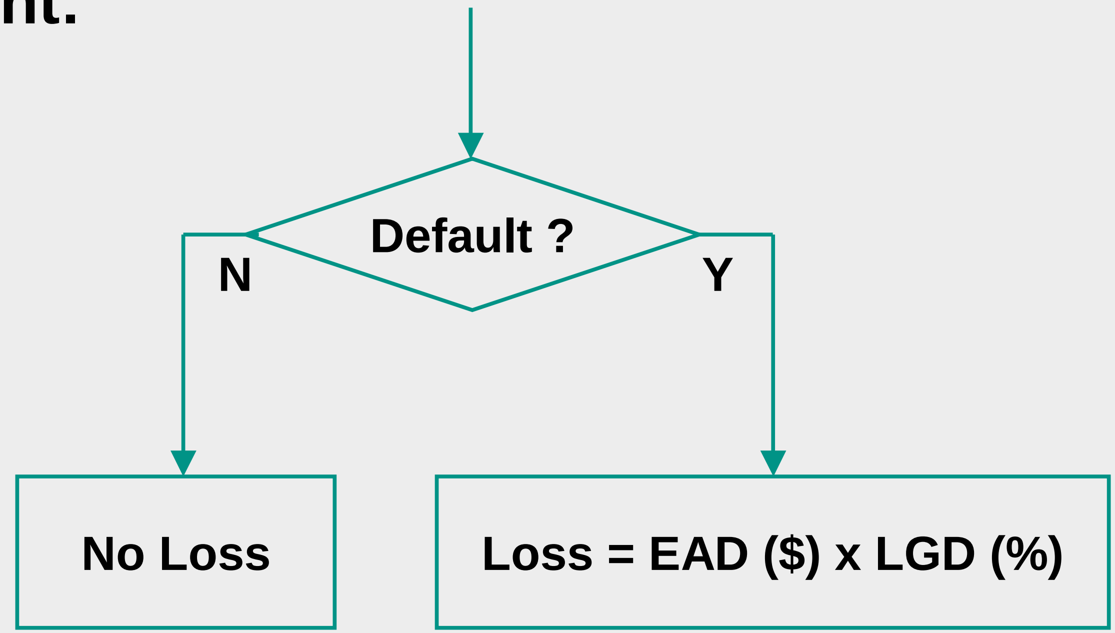
Credit Rating - Definition

Credit Rating = Assessment of risk on credit portfolios



Credit Rating – Expected Loss per client

Loss per client:



Expected Loss per client = PD (EAD x LGD)

Credit Rating – Expected Loss

EL = PD (EAD x LGD)

PD = Probability of Default

EAD = Exposure At Default

LGD = Loss Given Default

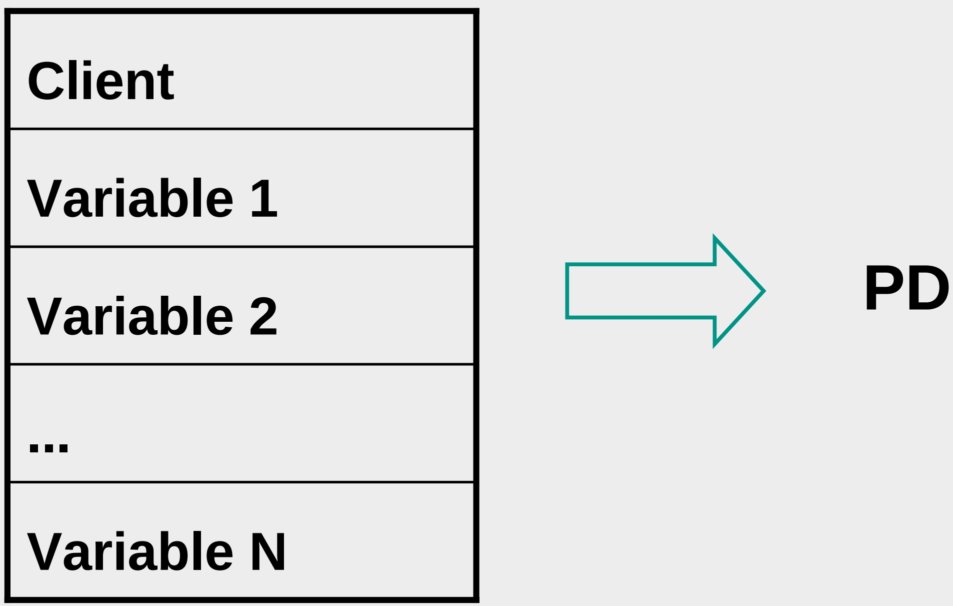
Credit Rating – Probability of Default

Probability of Default (PD)

= the probability that a client can not meet its repayment obligations between now and 1 year

Credit Rating – PD Model

PD calculation for each client



Credit Rating – Modeling steps

Construction of data set:

- **Select portfolio information from previous year**
- **Add default history from last 12 months**

Modeling technique: Binary logistic regression

Scoring: 5-fold cross validation

Credit Rating - Binary Logistic Regression

PD depends on variables (x_1 .. x_n) and parameters (β_0 .. β_n)

$$ln\left(\frac{PD}{1-PD}\right) = \beta_0 + \beta_1 x_1 + .. + \beta_n x_n$$

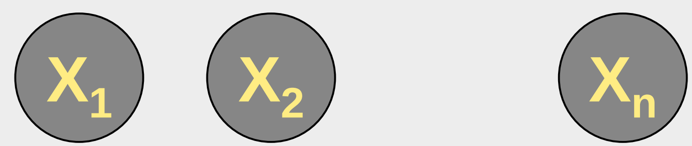
Calculation of (β_0 .. β_n) using maximum likelihood

Bayesian Networks - Definitions

A Bayesian Network (BN) is a probabilistic model of variables and their causal relations.

A BN is a directed acyclic graph where:

Variables are nodes..



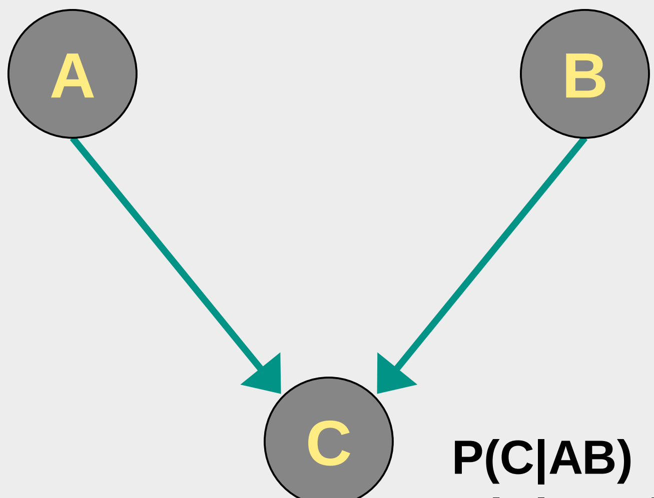
Causal relations are vertices...

Bayesian Networks – CPT

Conditional Probability Tables (CPT)

P(A)=0.02

P(B)=0.3



P(C|AB) = 0.95
P(C|A¬B) = 0.8
P(C|¬AB) = 0.1
P(C|¬A¬B) = 0.01

Bayesian Networks – Inference

- **Bayes' Theorem**

$$p(a|b)=\frac{p(b|a)p(a)}{p(b)}$$

- **Expansionrule**

$$p(a)=p(ab)+p(a\overline{b})=p(a|b)p(b)+p(a|\overline{b})p(\overline{b})$$

$$p(a)=p(abc)+p(ab\overline{c})+p(a\overline{b}c)+p(a\overline{b}\overline{c})$$

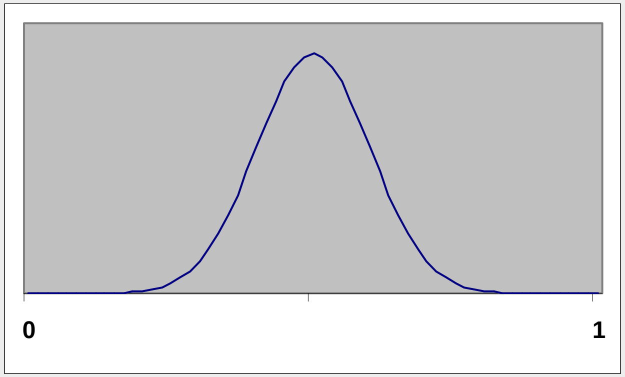
Bayesian Networks - Learning

θ = **probability of variable** X

$p(\theta)$ = **probability distribution of** θ

= **belief about** θ

Example of $p(\theta)$



Bayesian Networks – Belief Updating

Observation: O_i

The belief about θ **is now updated:**

$$p(\theta \mid O_i) = \begin{cases} c \cdot \theta \cdot p(\theta) & \text{if } \; O_i : X = 1 \\ c \cdot (1-\theta) \cdot p(\theta) & \text{if } \; O_i : X = 0 \end{cases}$$

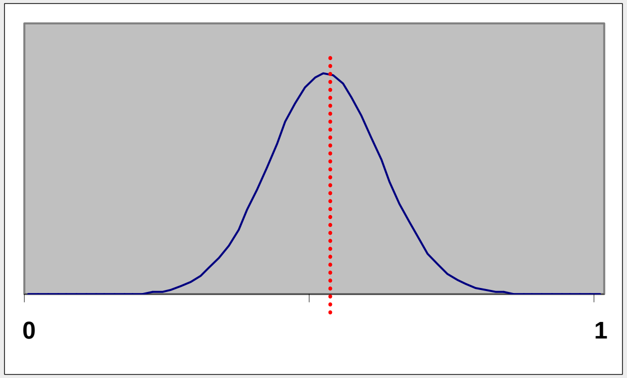
c **= normalization coefficient**

Bayesian Networks – Belief Updating

Observation O_i **:** X=1

Belief about θ **is updated**

Updated distribution $p(\theta | O_i) = p(\theta) \cdot \theta$



$$p(\theta | O_i)$$

Bayesian Networks – Beta distribution

Beta distribution with parameters α and β:

$$p(\theta \mid \alpha\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

α **= the number of observations** $X_i = 1$

β **= the number of observations** $X_i = 0$

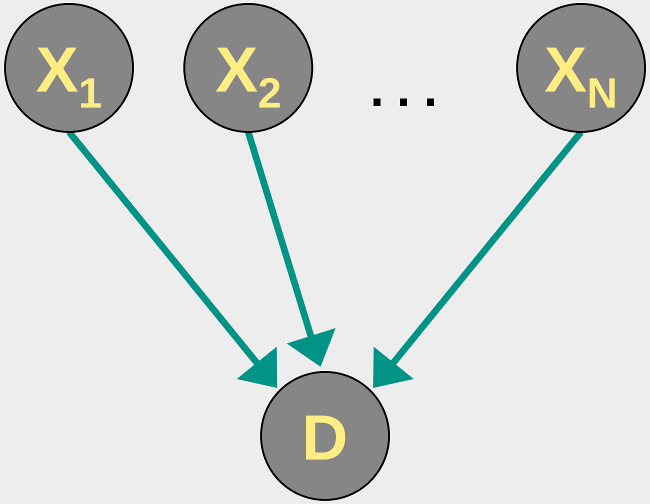
Property: after updating again Beta distribution

Bayesian Networks - Structures

Applied network structures in this research:

- **CPT model**
- **Naive Bayes model**
- **Noisy-MAX model**

Bayesian Networks – CPT model

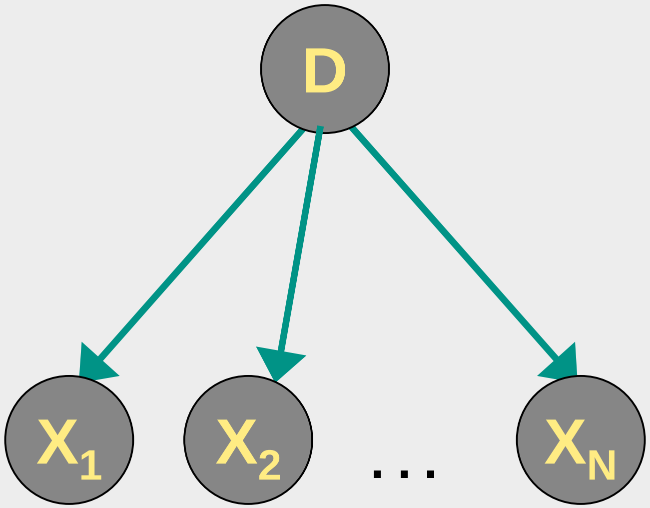


Size of CPT = $(s_p)^p$

s_p **= number of parent states**

p **= number of parents**

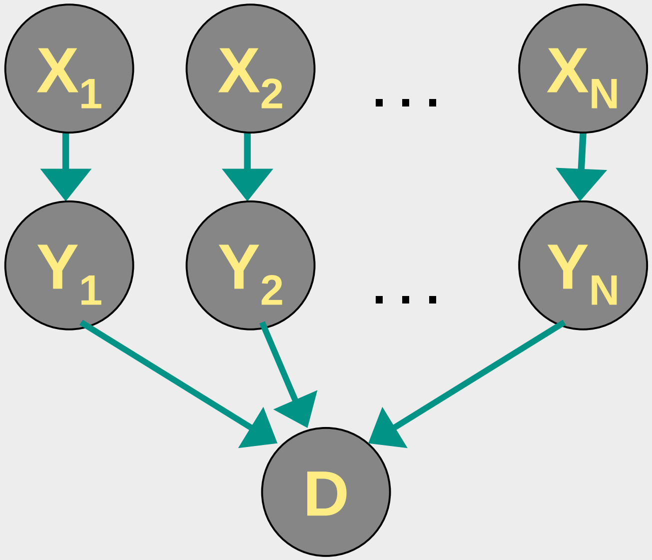
Bayesian Networks – Naive Bayes model



Unrealistic assumption:

Predictive variables dependent on effect variable.

Bayesian Networks – Noisy-MAX model



$P(Y_i \mid X_i) = c_i$

State of D **= logical MAX of** $Y_1..Y_N$

Assignment

- **Provide validated model for PD estimation for a credit portfolio. Logistic regression and Bayesian networks should be applied.**
- **Explore Bayesian network modeling and improve parameter learning techniques.**

Remainder of Presentation

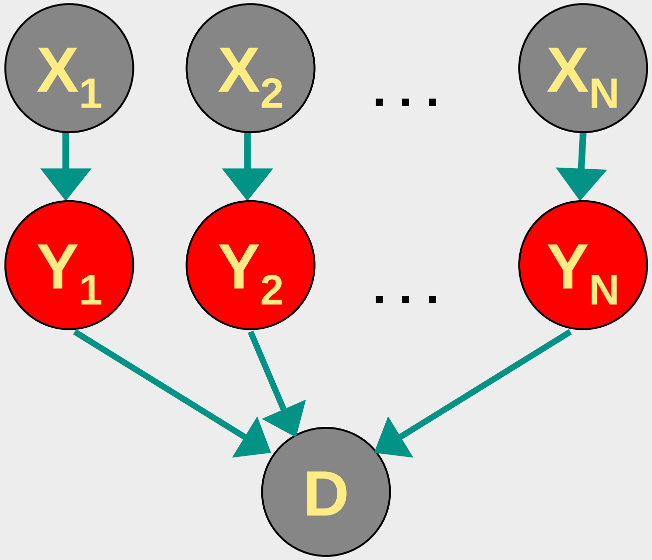
- Credit Rating
- Bayesian Networks
- Assignment
- **Bayesian Network Learning Research:**

  - **Noisy-MAX learning algorithm**

  - **Prior / Posterior learning**
- **Bayesian Networks in Credit Rating**
- **Conclusions**

Noisy-MAX Learning

Parameter learning based on observations not possible



$Y_1..Y_N$ **unknown**

Noisy-MAX Learning - Approach

- **Construct noisy-MAX CPT for node D, based on nodes $X_1..X_N$ and parameters $c_1..c_N$**
- **Learn normal CPT (not noisy-MAX) for node D, based on $X_1..X_N$**
- **Minimize difference between the two CPT's**

Noisy-MAX Learning – Noisy-MAX CPT

CPT for node Y_1

	$X_1 = 2$	$X_1 = 1$	$X_1 = 0$
$Y_1 = 2$	c_{122}	c_{121}	0
$Y_1 = 1$	c_{112}	c_{111}	0
$Y_1 = 0$	$1 - c_{112} - c_{122}$	$1 - c_{111} - c_{121}$	1

Noisy-MAX Learning – Noisy-MAX CPT

Cumulative CPT for node Y_1

	$X_1 = 2$	$X_1 = 1$	$X_1 = 0$
$Y_1 = 2$	c_{122}	c_{121}	0
$Y_1 = 1$	$c_{112} + c_{122}$	$c_{111} + c_{121}$	0
$Y_1 = 0$	1	1	1

$$p(Y_1 \geq y_1 \mid X_1 = x_1)$$

Noisy-MAX Learning – Noisy-MAX CPT

Transformation of cumulative into normal:

$$p(Y_1 = y_1 \mid X_1 = x_1)$$
$$= p(Y_1 \geq y_1 \mid X_1 = x_1) - p(Y_1 \geq y_1 + 1 \mid X_1 = x_1)$$

Noisy-MAX Learning – Noisy-MAX CPT

For MAX gates, the following holds:

$$
\begin{aligned}
& p(D \geq d \mid X_1 = x_1, .., X_N = x_N) \\
& \quad = p((Y_1 \geq d \mid X_1 = x_1) \vee (Y_2 \geq d \mid X_2 = x_2) \vee \ldots) \\
& \quad = 1 - \prod_{i=1}^{N} 1 - p(Y_i \geq d \mid X_i = x_i)
\end{aligned}
$$

→ Values for cumulative noisy-MAX CPT

Noisy-MAX Learning – Noisy-MAX CPT

Entries in noisy-MAX CPT defined as:

$$\theta_{ij} = \prod_{x_p^r \in \mathbf{X}} \sum_{k=1}^{j} q_{prk} - \prod_{x_p^r \in \mathbf{X}} \sum_{k=1}^{j-1} q_{prk}$$

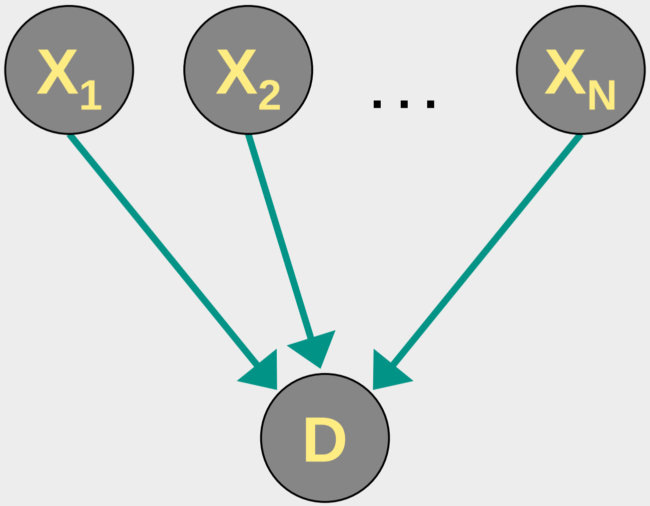
where

$$\theta_{ij} = p(D = d_j \,|\, \mathbf{X} = \mathbf{x_i})$$

$$q_{ijk} = p(Y_i = y_k \,|\, X_i = x_j)$$

Noisy-MAX Learning – Normal CPT

Normal CPT obtained by parameter learning



Noisy-MAX Learning – Distance

- **Euclidean Distance**

$$\sum_i \sum_j \left(\theta_{ij}^{CPT} - \theta_{ij}^{noisy-MAX}\right)^2$$

- **Kullback-Leibler Distance**

$$\sum_i \sum_j \theta_{ij}^{CPT} \, ln \frac{\theta_{ij}^{CPT}}{\theta_{ij}^{noisy-MAX}}$$

Noisy-MAX Learning – Optimization

Gradient Descent:

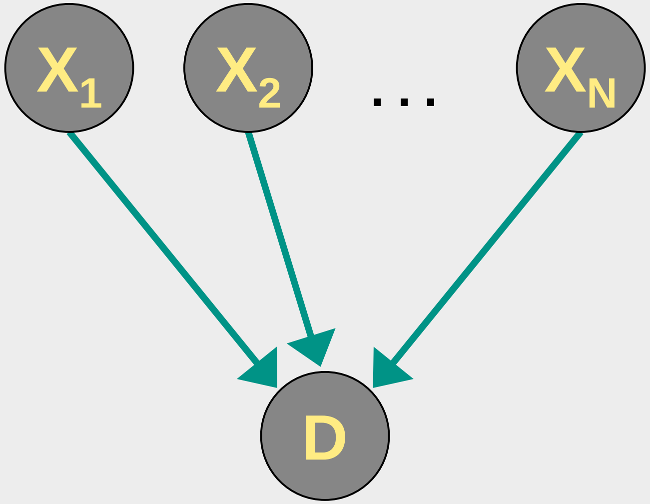
```
repeat
    for each noisy-MAX parameter c do
          c* = c +/- step
          d = CalculateDistance
          if d < d_min then
               d_min = d
               c = c*
          end if
    end for
until no distance reduction possible
```

Noisy-MAX Learning – Summary

- **Algorithm performs well**
- **Very fast**
- **Global minimum guaranteed**
- **EM performs better**
- **CPT not optimal starting point**

Prior / Posterior – Concept

CPT approach..



..but

- **prior is initial value in CPT**
- **posterior is prior updated by observations**

Prior / Posterior - Motive

CPT model has too many degrees of freedom

→ Rare entries have unreliable values

→ Resulting model is not stable

→ Setting priors can reduce instability

Prior / Posterior - Stages

1. **Prior assessment**
2. **Posterior learning**
3. **Merging**

Prior / Posterior – Posterior learning

CPT probabilities:

- **Beta distribution (binary variables)**

$$p(\theta \mid \alpha\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

- **Dirichlet distribution (multinomial variables)**

$$p(\theta \mid \alpha_1..\alpha_N) = \frac{\Gamma\left(\sum_{i=1}^{N} \alpha_i\right)}{\prod_{i=1}^{N} \Gamma(\alpha_i)} \prod_{i=1}^{N} \theta^{\alpha_i - 1}$$

Prior / Posterior – Posterior learning

Expectation of Beta distribution:

$$\int \theta \; Beta(\alpha\beta) \; d\theta = \frac{\alpha}{\alpha + \beta}$$

Expectation of Dirichlet distribution:

$$\int \theta_i \; Dir(\alpha_1..\alpha_N) \; d\theta_i = \frac{\alpha_i}{\sum_j \alpha_j}$$

Prior / Posterior – Posterior learning

CPT entries for variable X with states $x_1..x_N$:

$$\theta_{ij} = p(x_j \mid \mathbf{p}_i) = \frac{o(\mathbf{p}_i, x_j)}{\sum_{k=1}^{N} o(\mathbf{p}_i, x_k)}$$

$\mathbf{p}_i$ **= i-th combination of parent states**

$o(\mathbf{p}_i, x_j)$ **= number of observations $(\mathbf{p}_i, x_j)$ in the data set.**

Prior / Posterior – Prior assessment

Goal: Stable priors for every entry in the CPT

Solution:

1. **Build naive Bayes model**
2. **Learn parameters of CPT's**
3. **Perform inference to obtain** $p(d|x_1..x_N)$

Prior / Posterior – Prior assessment

Inference in naive Bayes:

$$
\begin{aligned}
p(d \mid x_1..x_N) \\
&= \frac{p(x_1..x_N \mid d)\cdot p(d)}{p(x_1..x_N)} \\
&= \frac{\prod_i p(x_i \mid d)\cdot p(d)}{\prod_i p(x_i)} \\
&= \frac{\prod_i p(x_i \mid d)\cdot p(d)}{\prod_i \left(p(x_i \mid d)\cdot p(d) + p(x_i \mid \bar{d})\cdot p(\bar{d})\right)}
\end{aligned}
$$

Prior / Posterior – Merging

Merging of priors and posteriors: weighting

$$\theta_{ij} = \frac{(w \cdot p_{ij}) + o(\mathbf{p}_i, x_j)}{w + \sum_{k=1}^{j} o(\mathbf{p}_i, x_k)}$$

weight w = 'number of observations' for the prior p_{ij}

Prior / Posterior – Summary

+ Very stable for rare event data

+ Naive Bayes priors guarantee high performance

+ Posterior CPT learning provides flexibility

BN in Credit Rating – Project results

- Construction of data set
- Development of logistic regression model
- BN tests with C++ tool
- Development of BN models
- Algorithm for Noisy-MAX
- Design & implementation of prior / posterior learning
- Comparison of BN and logistic regression

BN in Credit Rating - Comparison

Model	GINI score
Logistic Regression	78.3%
BN – CPT	55.6%
BN – Noisy-MAX	75.4%
BN – Naive Bayes	77.5%
BN – Prior / Posterior	77.3%

BN in Credit Rating - Comparison

Logistic regression

- **Favored by data selection process**
- **Optimization technique is by nature bound to yield good results**
- **Provides the right number of degrees of freedom**

BN in Credit Rating - Comparison

Bayesian Networks

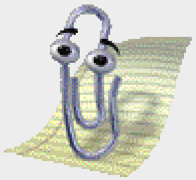
- Expert knowledge could not be incorporated
- CPT has too many degrees of freedom
- Naive Bayes performs well; apparently right number of degrees of freedom
- Logistic regression PD calculation and naive Bayes inference show similarities

Conclusions & Recommendations

BN is a very powerful modeling technique:

- **Expert knowledge combined with statistical data**
- **High acceptance due to transparency**
- **Good performance in diagnosis**

  (medical, trouble shooting, Microsoft Office Assistant)

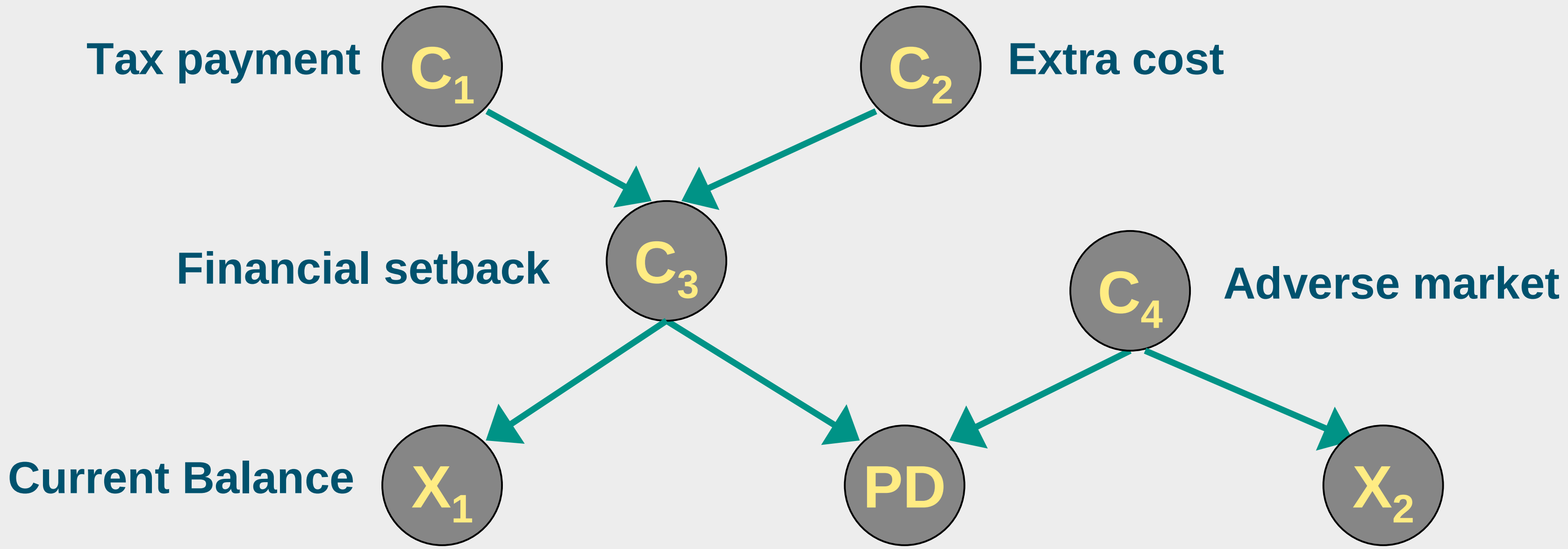


Conclusions & Recommendations

Improvements for BN results:

→ **Data selection process (stepwise BN)**

→ **High quality expert knowledge**



Conclusions & Recommendations

CPT approximation for Noisy-MAX learning

- **Noisy-MAX is very good alternative to CPT for rare event data**
- **CPT is probably not the best starting point for optimal parameters**

Conclusions & Recommendations

Prior / Posterior learning

- **Priors provide very stable networks**
- **High performance, promising technique**

→ **More testing on data**

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